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Neutral Current Effects in Bethe-Heitler Pair Production

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ABSTRACT

We consider Bethe-Heitler pair production in which the exchanged photon is replaced with a neutral weak intermediate boson. The interference between this and the pure Bethe-Heitler amplitudes contributes to an asymmetry between the lepton pairs and, in addition, the leptons acquire a finite longitudinal polarization. Both effects are calculated and numerical examples are given in the standard Weinberg-Salam model.

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I. INTRODUCTION

Weak neutral currents have been observed only in neutrino scattering.[1] Popular gauge theories of the weak and electromagnetic interactions predict that neutral currents should also show up in reactions not involving neutrinos, e.g., in $e^+e^- \rightarrow \mu^+\mu^-$, atomic transitions, etc. While the colliding beam experiment can best be done at high energy machines[2] now under construction (PEP and PETRA), ongoing experiments on atomic transitions have already reported[3] negative results, and their implications for gauge models have been analyzed.[4]

We have studied the effects of weak neutral currents in another neutrino-less experiment, viz. Bethe-Heitler pairs: $\gamma + N \rightarrow \ell^+ + \ell^- + X$. The pair $\ell^+\ell^-$ may be either electrons or muons.[5] By studying the interference between the weak and electromagnetic amplitudes we look for signals which are absent in pure electromagnetic Bethe-Heitler pair production. This is in the same spirit as neutral current calculations for $e^+e^- \rightarrow \mu^+\mu^-$, atomic transitions or $\ell^\pm N \rightarrow \ell^\pm X$,[6,7] the latter being closer to the type of calculation reported here. The virutal photon in the electromagnetic amplitude is replaced by a neutral intermediate vector boson Z_o having both vector and axial vector couplings to matter. Fig. 1 shows the Feynman diagrams involved in this calculation. A totally different set of diagrams also gives similar effects[8] which may be observable only if the Z_o (exchanged in the s-channel) is near its mass shell. If the Z_o is as heavy as gauge theories predict ($\gtrsim$ 75 GeV/c^2), this happens at far too large energies ($\sim$3 TeV). The present calculation is based on t-channel exchange and for all practical purposes we can let the boson mass $M_Z \to \infty$. A simple dimensional argument then tells us that the effects expected here are of order $G_F k^2/e^2$ where k^2 is the square of the momentum transferred to the target.

There are, of course, other pure electrodynamic processes for the photoproduction of lepton pairs. They do not give parity violating effects. Two photon exchange diagrams yield asymmetries very much like the ones derived here. These are odd under the interchange of the leptons and hopefully one can distinguish experimentally between the two by, e.g., their different behavior as one varies the photon energy. There are excellent reviews on Bethe-Heitler pairs and their background and we refer the reader to these.[9]

Compton-like diagrams, shown in Fig. 2, are expected to be smaller than the Bethe-Heitler diagrams which we calculate, in particular for large incoming photon energies. The s-channel diagrams involved in Compton-like pair production are highly damped since the intermediate hadron is far off mass-shell. In parton language, the struck quark must be very much off-mass-shell. We have therefore neglected these diagrams. Any calculation at best would be highly model dependent in contrast to Fig. 1.

In the next section we outline the derivation of the leptonic tensor and write down the hadronic tensor in a model-independent manner. By taking the appropriate products we obtain the totally differential cross section containing the square of the electromagnetic amplitude and its interference with the weak amplitude. In section III we adopt a particular model for the coupling of the intermediate boson. In section IV numerical results are presented on two effects: lepton polarization and asymmetry. Remarks and conclusions are collected in section V.

II. DERIVATION OF FORMULAS

A. Formalism

Our calculation of the process

$$\gamma(q) + N(P) \rightarrow \ell^{+}(p_{+}) + \ell^{-}(p_{-}) + X(P_{f}) \tag{1}$$

is based on the four Feynman diagrams shown in Fig. 1. Denoting the pure Bethe-Heitler amplitude corresponding to Figs. 1a and 1b by M_{EM}, and the weak amplitude corresponding to Figs. 1c and 1d by M_Z, we seek $\left|M_{EM}\right|^2 + 2\mathrm{Re}M^{*}_{EM}M_{Z}$. The interaction Hamiltonian we use is the following[10]

$$\mathscr{H} = e\bar{\psi}\gamma^{\mu}\psi A_{\mu} + \bar{\psi}\gamma^{\mu}(g_V - g_A\gamma_5)\psi Z_{\mu} - eJ^{\mu}_{EM}A_{\mu} - g_h J^{\mu}_{W} Z_{\mu} \quad . \tag{2}$$

We will first calculate the weak amplitude

$$M_Z = \frac{-ieg_V g_h}{k^2 - M_Z^{\,2}} \epsilon^{\sigma} J^{\rho}_{W} \bar{u}(p_-) \left\{ \Gamma^{-}_{\sigma\rho} - \Gamma^{+}_{\rho\sigma} \right\} (1 - g_r\gamma_5) v(p_+) \quad , \tag{3}$$

where

$$\Gamma^{\pm}_{\alpha\beta} = \gamma_\alpha \frac{1}{\not{p}_\pm - \not{q}} \gamma_\beta = \frac{-1}{2q \cdot p_\pm} \gamma_\alpha (\not{p}_\pm - \not{q}) \gamma_\beta \quad . \tag{4}$$

$\epsilon^\sigma = \epsilon^\sigma(q)$ is the polarization four-vector of the incoming photon, and we have used momentum conservation $p_- - k = q - p_+$. All lepton masses have been dropped, and we have defined $g_r = g_A / g_V$. By obvious replacements in equation (3) we obtain the electromagnetic amplitude

$$M_{EM} = \frac{-ie^3}{k^2} \epsilon^\sigma J^\rho_{EM} \bar{u}(p_-) \left\{ \Gamma^-_{\sigma\rho} - \Gamma^+_{\rho\sigma} \right\} v(p_+) \quad . \tag{5}$$

In terms of the leptonic currents j_ρ and j_ρ^5, where

$$j_\rho^5 = \epsilon^\sigma \bar{u}(p_-) \left\{ \Gamma^-_{\sigma\rho} - \Gamma^+_{\rho\sigma} \right\} (1 - g_r \gamma_5) v(p_+) \quad , \tag{6}$$

and

$$j_\rho = j_\rho^5 (g_r = 0) \quad ,$$

we immediately obtain

$$\left| M_{EM} \right|^2 = \frac{e^6}{k^4} j_{\rho'} j_\rho^* J^{\rho'}_{EM} J^{*\rho}_{EM} \tag{7}$$

and

$$2R_e M^*_{EM} M_Z = \frac{e^4 g_V g_h}{k^2(k^2 - M_Z^2)} \left\{ j_{\rho'} j^{*5}_{\rho} J^{\rho'}_{EM} J^{*\rho}_{W} + j^{*}_{\rho} j^{5}_{\rho'} J^{*\rho}_{EM} J^{\rho'}_{W} \right\}$$

$$= \frac{e^4 g_V g_h}{k^2(k^2 - M_Z^2)} j_{\rho'} j^{*5}_{\rho} \left\{ J^{*\rho}_{W} J^{\rho'}_{EM} + J^{\rho'}_{W} J^{*\rho}_{EM} \right\} \tag{8}$$

where we have used the relation $j^{*}_{\rho} j^{5}_{\rho'} = j_{\rho'} j^{*5}_{\rho}$.

In the next section we calculate the leptonic tensors $j_{\rho'} j^{*}_{\rho}$ and $j_{\rho'} j^{*5}_{\rho}$. In section IIC we give the hadronic tensors $J^{\rho'}_{EM} J^{*\rho}_{EM}$ and $J^{*\rho}_{W} J^{\rho'}_{EM} + J^{\rho'}_{W} J^{*\rho}_{EM}$, and in section IID we give the final result for the differential cross section. In section IIE we write down the expressions for the polarization and the asymmetry of the leptons. The various formulas are discussed in section IIF.

B. The Leptonic Tensors

We first calculate $j_{\rho'} j^{*5}_{\rho}$ since $j_{\rho'} j^{*}_{\rho}$ can be obtained from it simply by setting $g_r = 0$. We average over the photon polarization but keep the lepton helicities[11] in the tensor $j_{\rho'} j^{*5}_{\rho}$:

$$\frac{1}{2} \sum_{\lambda_\gamma} j_{\rho'} j^{*5}_{\rho} = -\frac{1}{8(2m)^2} \left\{ \alpha S_{\rho\rho'} + \beta S^5_{\rho\rho'} \right\} \tag{9}$$

where

$$\alpha = 1 - \lambda_+ \lambda_- + (\lambda_+ - \lambda_-) g_r \ , \ \beta = \lambda_- - \lambda_+ - (1 - \lambda_+ \lambda_-) g_r \ .$$

The traces are

$$S_{\rho\rho'} = \mathrm{Tr}\left\{\not{p}_+\left[\Gamma^-_{\rho\sigma} - \Gamma^+_{\sigma\rho}\right]\not{p}_-\left[\Gamma^{-\sigma}_{\rho'} - \Gamma^{+\sigma}_{\rho'}\right]\right\} \tag{10a}$$

and

$$S^5_{\rho\rho'} = \mathrm{Tr}\left\{\gamma_5\not{p}_+\left[\Gamma^-_{\rho\sigma} - \Gamma^+_{\sigma\rho}\right]\not{p}_-\left[\Gamma^{-\sigma}_{\rho'} - \Gamma^{+\sigma}_{\rho'}\right]\right\} \quad . \tag{10b}$$

One can prove that $S_{\rho\rho'}$ and $S^5_{\rho\rho'}$ satisfy the relations

$$S_{\rho\rho'}(p_+, p_-) = S_{\rho'\rho}(p_+, p_-) = S_{\rho\rho'}(p_-, p_+) \tag{11a}$$

and

$$S^5_{\rho\rho'}(p_+, p_-) = -S^5_{\rho'\rho}(p_+, p_-) = -S^5_{\rho\rho'}(p_-, p_+) \quad . \tag{11b}$$

These relations can be used to simplify the calculation of the traces. We shall express the result in terms of the following variables introduced by Drell and Walecka:[12]

$$\ell = q + k = p_+ + p_- \ , \quad \Delta = p_- - p_+ \ ,$$

$$x_1 = 2q \cdot p_- \ , \quad x_2 = 2q \cdot p_+ \ , \quad x_3 = \frac{P \cdot q}{M_T} \ , \tag{12}$$

$$x_4 = \frac{P \cdot \Delta}{M_T} \ , \quad x_5 = \frac{P \cdot k}{M_T} \ , \quad x_6 = k^2 \ ,$$

where M_T is the target mass. Then,

$$\tfrac{1}{2} S_{\rho\rho'} = M_1 g_{\rho\rho'} + M_2 \ell_\rho \ell_{\rho'} + M_3 \Delta_\rho \Delta_{\rho'} + M_4 (\Delta_\rho \ell_{\rho'} + \Delta_{\rho'} \ell_\rho)$$
$$+ M_5 (\ell_\rho k_{\rho'} + \ell_{\rho'} k_\rho) + M_6 (\Delta_\rho k_{\rho'} + \Delta_{\rho'} k_\rho) \qquad (13)$$

and

$$\frac{1}{2i} S^5_{\rho\rho'} = N_1 \epsilon_{\rho\rho'\mu\nu} p_-^\mu p_+^\nu + N_2 \epsilon_{\rho\rho'\mu\nu} q^\mu p_+^\nu + N_3 \epsilon_{\rho\rho'\mu\nu} q^\mu p_-^\nu \quad . \qquad (14)$$

In these equations $M_1, \ldots N_3$ are given by the following:

$$M_1 = \frac{4}{x_1 x_2} \left\{ x_6^{\,2} + x_6 (x_1 + x_2) + \tfrac{1}{2} (x_1^{\,2} + x_2^{\,2}) \right\} ,$$

$$M_2 = 4x_6 / x_1 x_2 \ , \quad M_3 = M_2 \ , \quad M_4 = 0 \ ,$$

$$M_5 = -2(x_1 + x_2 + 2x_6)/x_1 x_2 \ , \quad M_6 = 2(x_1 - x_2)/x_1 x_2 \ , \qquad (15)$$

$$N_1 = 2M_5 \ , \quad N_2 = 4(x_1 + x_6)/x_1 x_2 \ , \quad N_3 = -4(x_2 + x_6) x_1 x_2 \quad .$$

One can check that our expression for $S_{\rho\rho'}$ agrees with that of reference 12 when the lepton mass is set equal to zero. Our additional terms proportional to k_ρ are necessary for a gauge invariant tensor, i.e., our $S_{\rho\rho'}$ satisfies

$$k_\rho S^{\rho\rho'} = 0 \quad . \qquad (16)$$

Since we have dropped the lepton masses, a similar relation holds for $S^5_{\rho\rho'}$:

$$k_\rho S^{5\rho\rho'} = 0 \quad . \tag{17}$$

Of course, one may drop the term proportional to k_ρ (M_5 and M_6) if all relevant hadronic currents are conserved.[12] We chose to keep these terms to allow for the possibility that the hadronic currents (vector and axial vector) may not be conserved, and also to make the simplification later in the hadronic tensor where we shall drop all terms proportional to k_ρ by virtue of (16) and (17).

As mentioned earlier, we simply put $g_r = 0$ to obtain the pure Bethe-Heitler amplitude. Therefore

$$\tfrac{1}{2}\sum_{\lambda_\gamma} j_{\rho'} j^*_\rho = \tfrac{1}{2}\sum_{\lambda_\gamma} (j_{\rho'} j^{*5}_\rho)_{g_r = 0} = -\frac{1}{8(2m)^2}\left\{\alpha_0 S_{\rho\rho'} + \beta_0 S^5_{\rho\rho'}\right\} , \tag{18}$$

where $\alpha_0 = 1 - \lambda_+\lambda_-$, $\beta_0 = \lambda_- - \lambda_+$, and $S_{\rho\rho'}$ and $S^5_{\rho\rho'}$ are the same tensors given above by Eqs. (13) and (14).

C. The Hadronic Tensors

The structure of the hadronic tensor involving $J^{\rho'}_{EM} J^{*\rho}_{EM}$ is well known and, averaging over the spin of the target and summing over final state variables, can be written in the form

$$W^{\rho\rho'}(-k, P) = \overline{\sum_{s_T}} \sum_{s_f, P_f} J^{\rho'}_{EM} J^{*\rho}_{EM} \delta^4(P - k - P_f)$$

$$= -W_1\left(g^{\rho\rho'} - \frac{k_\rho k_{\rho'}}{k^2}\right) + \frac{W_2}{M_T^2}\left(P - \frac{P\cdot k}{k^2} k\right)^\rho \left(P - \frac{P\cdot k}{k^2} k\right)^{\rho'} \tag{19a}$$

which becomes

$$W^{\rho\rho'}(-k, P) = -g^{\rho\rho'} W_1 + \frac{P^\rho P^{\rho'}}{M_T^2} W_2 \tag{19b}$$

when the terms proportional to k_ρ are dropped.

For the tensor describing the interference of the weak and electromagnetic amplitudes at the hadronic vertex we follow the notation of reference 6 and write

$$R_{\rho\rho'}(-k, P) = \overline{\sum_{s_T}} \sum_{s_f, P_f} (J^{\rho'}_{EM} J^{*\rho}_{EM} + J^{\rho'}_{W} J^{*\rho}_{EM}) \delta^4(P - k - P_f)$$

$$= -R_1\left(g_{\rho\rho'} - \frac{k_\rho k_{\rho'}}{k^2}\right) + \frac{R_2}{M_T^2}\left(P - \frac{P\cdot k}{k^2} k\right)^\rho \left(P - \frac{P\cdot k}{k^2}\right)^{\rho'} \tag{20a}$$

$$+ \frac{iR_3}{2M_T^2} \epsilon_{\rho\rho'\alpha\beta} P^\alpha k^\beta - \frac{R_5}{M_T^2}\left(P_\rho k_{\rho'} + P_{\rho'} k_\rho - 2\frac{P\cdot k}{k^2} k_\rho k_{\rho'}\right)$$

which becomes simply

$$R_{\rho\rho'}(-k, P) = -g_{\rho\rho'} R_1 + \frac{P_\rho P_{\rho'}}{M_T^2} R_2 + i\,\epsilon_{\rho\rho'\alpha\beta} \frac{P^\alpha k^\beta}{2M_T^2} R_3 \tag{20b}$$

after dropping the terms proportional to k_ρ.

The structure functions R_1, R_2, R_3, and, of course, W_1 and W_2 are functions of k^2 and $\nu = -x_5$. νW_2 and presumably W_1, R_1, νR_2 and νR_3 scale in the variable x where

$$x = \frac{-k^2}{2M_T \nu} = \frac{x_6}{2M_T x_5} .$$

D. The Differential Cross Section

We shall next calculate the totally differential cross section. Defining

$$d\phi_\pm = \frac{d^3 p_\pm}{2E_\pm}$$

we find that

$$\frac{d\sigma(\lambda_+, \lambda_-)}{d\phi_+ d\phi_-} = \frac{-e^6 M_T}{16(2\pi)^5 k^4 P \cdot q} \left\{ (\alpha_0 S_{\rho\rho'} + \beta_0 S^5_{\rho\rho'}) W^{\rho\rho'} \right.$$
$$\left. + \frac{k^2 g_V g_h}{e^2 (k^2 - M_Z^2)} (\alpha S_{\rho\rho'} + \beta S^5_{\rho\rho'}) R^{\rho\rho'} \right\} . \tag{21}$$

We note that (see Eq. (14)) $S^5_{\rho\rho'}$ is antisymmetric under $\rho \leftrightarrow \rho'$, so that $S^5_{\rho\rho'} W^{\rho\rho'} = 0$. Similarly, only the last term, R_3, contributes to $S^5_{\rho\rho'} R^{\rho\rho'}$. The final result is:

$$\frac{d\sigma(\lambda_+, \lambda_-)}{d\phi_+ d\phi_-} = \frac{(e^2/4\pi)^3}{\pi^2 x_1 x_2 x_3 x_6^2} \Bigg\{ \alpha_0 (L_1 W_1 - L_2 W_2)$$
$$+ \frac{g_V g_h x_6}{e^2 (x_6 - M_Z^2)} \left[\alpha (L_1 R_1 - L_2 R_2) + \frac{\beta}{2M_T} L_3 R_3 \right] \Bigg\} \tag{22}$$

where

$$L_1 = \frac{x_1 x_2}{8} S^\rho_\rho = x_1^2 + x_2^2 + 2x_6(x_1 + x_2 + x_6) \quad ,$$

$$L_2 = \frac{x_1 x_2}{8} \frac{P^\rho P^{\rho'}}{M_T^2} S_{\rho\rho'} = (x_6 - x_5^2)(x_1 + x_2 + x_6) + \tfrac{1}{2}(x_1^2 + x_2^2)$$
$$- x_3 x_5 (x_1 + x_2) + x_6 (x_3^2 + x_4^2) + x_4 x_5 (x_1 - x_2) \quad , \tag{23}$$

$$L_3 = \frac{-i x_1 x_2}{8M_T} \epsilon_{\rho\rho'\alpha\beta} S^{5\rho\rho'} P^\alpha k^\beta = (x_1 - x_2)\left[x_5(x_1 + x_2 + x_6) - x_3 x_6\right]$$
$$+ x_4 x_6 (x_1 + x_2 + 2x_6) \quad .$$

If the polarizations of the final leptons are not measured, one must sum over both helicities and the resulting cross section is

$$\frac{d\sigma(p_+, p_-, \ldots)}{d\phi_+ d\phi_-} = \sum_{\lambda_+, \lambda_-} \frac{d\sigma(\lambda_+, \lambda_-)}{d\phi_+ d\phi_-} = \frac{4(e^2/4\pi)^3}{\pi^2 x_1 x_2 x_3 x_6^2} \Bigg\{ L_1 W_1 - L_2 W_2$$
$$+ \frac{g_V g_h x_6}{e^2 (x_6 - M_Z^2)} \left[L_1 R_1 - L_2 R_2 - g_r \frac{L_3 R_3}{2M_T} \right] \Bigg\} \quad . \tag{24}$$

E. The Polarization and Asymmetry of Final Leptons

We will assume that the polarization of only one of the leptons, ℓ^+ or ℓ^-, is measured. Note that the pure Bethe-Heitler diagrams predict a correlation between these polarizations in the case of a simultaneous measurement, viz. $\lambda_+ = -\lambda_-$, but do not constrain the values of λ_+ or λ_- independently ($<\lambda_+> = <\lambda_-> = 0$). The interference between the weak and electromagnetic amplitudes will cause a net polarization:

$$
\begin{aligned}
<\lambda_+> &= \frac{\sum_{\lambda_-} \left\{ d\sigma(\lambda_+ = 1,\ \lambda_-) - d\sigma(\lambda_+ = -1,\ \lambda_-) \right\}}{\sum_{\lambda_-, \lambda_+} d\sigma(\lambda_+,\ \lambda_-)} \\
&= \frac{g_h x_6 \left[g_A (L_1 R_1 - L_2 R_2) - \frac{g_V}{2M_T} L_3 R_3 \right]}{e^2 (x_6 - M_Z^2)(L_1 W_1 - L_2 W_2)} \quad .
\end{aligned}
\tag{25}
$$

Another signal which comes from the weak amplitude is an asymmetry under the interchange of ℓ^+ and ℓ^-:

$$
\begin{aligned}
A &= \frac{d\sigma(p_+,\ p_-,\ \ldots) - d\sigma(p_-,\ p_+,\ \ldots)}{d\sigma(p_+,\ p_-,\ \ldots) + d\sigma(p_-,\ p_+,\ \ldots)} \\
&= \frac{-g_A g_h x_6 L_3 R_3}{2M_T e^2 (x_6 - M_Z^2)(L_1 W_1 - L_2 W_2)} \quad .
\end{aligned}
\tag{26}
$$

There are, of course, also deviations from the pure Bethe-Heitler cross section even for symmetric pairs, especially for lage values of $|k^2|$:

$$\left(\frac{d\sigma_{\text{B.-H. + Weak}}}{d\sigma_{\text{B.-H.}}}\right)_{\text{sym.}} = 1 + \frac{g_V g_h x_6 (L_1 R_1 - L_2 R_2)}{e^2 (x_6 - M_Z^2)(L_1 W_1 - L_2 W_2)} \quad . \tag{27}$$

F. Discussion of Formulas

If the polarization of only ℓ^-, rather than ℓ^+, is to be measured, then one simply must change the overall sign of Eq. (25), i.e., $\langle\lambda_-\rangle = -\langle\lambda_+\rangle$. This is a consequence of dropping the lepton mass and the subsequent γ_5-invariance of $\mathscr{H}$(leptonic).

For symmetric pairs, $L_3 = 0$, and Eq. (25) simplifies to

$$\langle\lambda_+\rangle_{\text{sym.}} = -\langle\lambda_-\rangle_{\text{sym.}} = \frac{g_A g_h x_6 (L_1 R_1 - L_2 R_2)}{e^2 (x_6 - M_Z^2)(L_1 W_1 - L_2 W_2)} \quad . \tag{28}$$

Under rather general assumptions (see below) $R_1/R_2 = W_1/W_2$ in which case Eqs. (27) and (28) become almost independent of the lepton kinematics and are given by

$$\left(\frac{d\sigma_{\text{B.-H. + Weak}}}{d\sigma_{\text{B.-H.}}}\right)_{\text{sym.}} = 1 + \frac{g_V g_h x_6 R_2}{e^2 (x_6 - M_Z^2) W_2} \tag{29}$$

and

$$\langle\lambda_+\rangle_{\text{sym.}} = -\langle\lambda_-\rangle_{\text{sym.}} = \frac{g_A g_h x_6 R_2}{e^2 (x_6 - M_Z^2) W_2} \tag{30}$$

respectively. Since in most gauge models $g_V g_h \sim g_A g_h \sim M_Z^{\;2} G_F$ and $R_2 \sim W_2$, when $|x_6| = |k^2| \ll M_Z^{\;2}$ we recover the crude estimate made in the Introduction of the magnitude of the effects, namely $G_F k^2/e^2$.

III. THE STRUCTURE FUNCTIONS AND NEUTRAL CURRENT MODEL

To make numerical estimates of the sizes of the effects discussed above we need a specific model for the currents entering in the Hamiltonian Eq. (2). We shall proceed in three steps to define such a model.

First we shall assume that the Callan-Gross relation holds not only between W_1 and W_2, but also between R_1 and R_2:

$$\nu W_2 = 2M_T x W_1 \tag{31a}$$

$$\nu R_2 = 2M_T x R_1 \quad . \tag{31b}$$

Then the following combinations of structure functions simplify, becoming

$$L_1 W_1 - L_2 W_2 = -x_5 W_2 L_{12} \tag{32a}$$

and

$$L_1 R_1 - L_2 R_2 = -x_5 R_2 L_{12} \tag{32b}$$

where

$$L_{12} = (x_1{}^2 + x_2{}^2)\left(\frac{x_5}{x_6} + \frac{1}{2x_5}\right) + (x_1 + x_2 + x_6)\left(\frac{x_6}{x_5} + x_5\right)$$
$$+ \frac{x_6(x_3{}^2 + x_4{}^2)}{x_5} + x_4(x_1 - x_2) - x_3(x_1 + x_2) \quad . \tag{33}$$

Secondly we assume that the structure functions are adequately described by a quark-parton model, i.e., assume

$$J^\mu_{EM} = \sum_{\text{quarks}} Q_i \bar{q}_i \gamma^\mu q_i \tag{34a}$$

and

$$J^\mu_W = \sum_{\text{quarks}} \bar{q}_i \gamma^\mu (a_i - b_i \gamma_5) q_i \quad . \tag{34b}$$

Then

$$\nu W_2 = x \sum_{\text{quarks}} Q_i{}^2 \left[p_i(x) + p_{\bar{i}}(x)\right] \quad ,$$

$$\nu R_2 = 2x \sum_{\text{quarks}} Q_i a_i \left[p_i(x) + p_{\bar{i}}(x)\right] \quad ,$$

and

$$\nu R_3 = -2 \sum_{\text{quarks}} Q_i b_i \left[p_i(x) - p_{\bar{i}}(x)\right] \tag{35}$$

where $p_i(x)$ and $p_{\bar{i}}(x)$ are the usual probability functions for quarks and antiquarks, respectively.

Finally, we shall use the standard Weinberg-Salam model[13] in which case the coupling constants are the following:

$$g_V = \frac{e}{2\sin 2\theta_W}(4\sin^2\theta_W - 1)\ ;\ g_A = \frac{-e}{2\sin 2\theta_W}\ ;\ g_h = \frac{-e}{\sin 2\theta_W}\ ;$$

$$a_u = a_c = \frac{1}{2} - \frac{4}{3}\sin^2\theta_W;\ a_d = a_s = -\frac{1}{2} + \frac{2}{3}\sin^2\theta_W\ ; \tag{36}$$

$$b_u = b_c = \frac{1}{2}\ ;\ b_d = b_s = -\frac{1}{2}\quad .$$

The mass of the vector boson Z^0 is given by

$$M_Z^{\ 2} = \frac{e^2}{\sqrt{2}\,G_F(\sin 2\theta_W)^2}\quad . \tag{37}$$

The only remaining free parameter is the Weinberg angle θ_W and for numerical calculations below we shall choose the experimentally favored value[1] $\sin^2\theta_W = 0.3$.

The probability functions $p_i(x)$ and $p_{\bar{i}}(x)$ can be parametrized using either deep inelastic electron or neutrino scattering data. We shall use the parametrization given by Barger and Phillips.[14] For a proton target,

$$p_s = p_{\bar{s}} = p_{\bar{u}} = p_{\bar{d}} = s = \frac{0.145}{x}(1-x)^9\quad , \tag{38a}$$

$$p_u = \frac{1}{\sqrt{x}}\left\{0.594(1-x^2)^3 + 0.461(1-x^2)^5 + 0.621(1-x^2)^7\right\} + s\quad , \tag{38b}$$

$$p_d = \frac{1}{\sqrt{x}}\left\{0.072(1-x^2)^3 + 0.206(1-x^2)^5 + 0.621(1-x^2)^7\right\} + s\quad . \tag{38c}$$

The probability functions for a neutron target are obtained by interchanging p_u and p_d in the above equation.

IV. NUMERICAL RESULTS

For $|k^2| \ll M_Z^{\,2}$ in the Weinberg-Salam model[13] $\langle\lambda_+\rangle$ and A are given by the following simple expressions:

$$A = -\frac{G_F}{\sqrt{2}}\,\frac{1}{e^2}\,\frac{xR_3L_3}{W_2L_{12}} \quad , \tag{39}$$

and

$$\begin{aligned}\langle\lambda_+\rangle &= \frac{-G_F}{\sqrt{2}}\,\frac{1}{e^2}\,\frac{x_6R_2}{W_2} + (1 - 4\sin^2\theta_W)A \\ &\approx \frac{|k^2|R_2/W_2}{115\ \mathrm{GeV}^2}\,\% + (1 - 4\sin^2\theta_W)A \quad .\end{aligned} \tag{40}$$

In Fig. 3 we plot νW_2, R_2/W_2, and xR_3/W_2 all of which are functions of x only. We have chosen an isoscalar target. These quantities serve to illustrate the hadronic depdndence of the neutral current effects independently of the lepton pair kinematical configuration which is contained in L_{12} and L_3. In fact, these same functions also appear in other processes where the weak neutral and the electromagnetic currents can interfere, e.g., $\ell^\pm + p \to \ell^\pm + X$ and $e^+ + e^- \to p + X$.

The kinematical variables for the leptons in the laboratory system are defined in Fig. 4. We shall present numerical results for the following two kinematical configurations for purposes of illustration:

$$\theta_+ = 24^\circ \ , \ \theta_- = 2^\circ \ , \ \phi = 180^\circ$$

and

$$\theta_+ = 15^\circ \ , \ \theta_- = 5^\circ \ , \ \phi = 180^\circ \ .$$

For each of these configurations we consider two photon energies, $E_\gamma = 150$ GeV and $E_\gamma = 200$ GeV. For these four cases we have plotted the asymmetry A and the polarization $<\lambda_+>$ in Figs. 5, 6, 7 and 8 choosing two values of E_+, viz. $E_+ = 5$ GeV and $E_+ = 10$ GeV and taking E_- in the range 10 GeV $< E_- <$ 100 GeV.

V. DISCUSSION

From Eqs. (39) and (40) it is evident that most of the dependence on the leptonic kinematical configuration enters through the asymmetry A which involves L_3/L_{12}. For the completely symmetric configuration, A vanishes and $<\lambda_+>$ is simply proportional to k^2 and involves x only through the function R_2/W_2.

From Figs. 5, 6, 7 and 8 one sees that both the polarization $<\lambda_+>$ and the asymmetry A are of the order of 1% to 5%. The effects tend to be smaller when the kinematical configuration is more symmetric, as one might expect. From Figs. 5 and 6 one sees that for this kinematical configuration both $<\lambda_+>$ and A are roughly proportional to E_γ and also depend linearly on E_+ and E_-. Figs. 7 and 8 illustrate that for this more symmetric kinematical configuration $\langle\lambda_+\rangle$ and A remain crudely proportional to E_γ and approximately linear in E_+; however, the dependence on E_- is more complicated. Apparently $\langle\lambda_+\rangle$ and A vary linearly with E_+ and E_- only for $E_\pm \ll E_\gamma$.

It is clear that observation of a nonvanishing lepton polarization is conclusive proof of parity violation in photoproduction of lepton pairs. However, other purely electromagnetic amplitudes interfering with the Bethe-Heitler amplitude will also contribute to the asymmetry A between ℓ^+ and ℓ^-. The energy dependence, however, of the asymmetry arising from higher order electromagnetic effects and the neutral current effect calculated here will differ considerably. This is expected on the basis of the simple dimensional argument that the weak amplitude will involve the Fermi coupling $G_F \sim 10^{-5}/\mathrm{GeV}^2$, hence the asymmetry $\sim G_F E^2$, while the pure electromagnetic asymmetry is mildly, if at all, dependent on energy.

A very important advantage in looking for neutral current effects in charged lepton + hadron systems like we have considered here lies in the fact that the signals survive even in the case when the neutral current coupling to the charged leptons conserves parity ($g_A = 0$). That this might be the case is suggested by the negative results of the atomic parity violation experiments.[3,4] In such low energy experiments parity violation at the hadronic vertex is much too small to be detected, so that one may plausibly argue that the failure to observe the predicted rotation of the plane of the polarized light in these atomic experiments indicate $g_A = 0$.

Certainly our numerical results in section IV are somewhat model-dependent. However, the Weinberg-Salam model does enjoy a certain amount of experimental support and has the virtue that it is a one-parameter theory with that one parameter, viz. $\sin^2 \theta_W$, fairly well determined from the analysis of neutral current data. Of course, it cannot, in its simplest form, accomodate $g_A = 0$. In addition, we had to choose specific parton distribution functions $p_i(x)$ and $p_{\bar{i}}(x)$. Since only the ratios of these functions appear in the quantities of interest, we expect that a different choice would not significantly affect the final results.

In conclusion, we have shown that approximately 1% - 5% neutral current effects can be expected in the photoproduction of lepton pairs at presently available beam energies of 100 - 200 GeV. The signals have been found to be most pronounced for highly asymmetric pairs. We emphasize that nonvanishing polarizations are expected in any theory where <u>either</u> the leptons <u>or</u> the hadrons <u>or both</u> have a $\gamma_5\gamma_\mu$ coupling to the weak boson.

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$$v(p_+)\bar{v}(p_+) \to \tfrac{1}{2}(1 - \lambda_+\gamma_5)\,\frac{1}{2m}\,p_+ \quad ,$$

and

$$u(p_-)\bar{u}(p_-) \to \tfrac{1}{2}(1 + \lambda_-\gamma_5)\,\frac{1}{2m}\,p_- \quad ,$$

where m = lepton mass.

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FIGURE CAPTIONS

Fig. 1: Feynman diagrams for the electromagnetic contributions (a and b) and the weak contributions (c and d) to the photoproduction of lepton pairs.

Fig. 2: Compton-like electromagnetic (a) and weak (b) amplitudes for lepton pair photoproduction.

Fig. 3: The combination of hadronic vertex functions νW_2, R_2/W_2, and xR_3/W_2 which appear in the present calculations. $x = -k^2/2M_T\nu$ is the usual scaling variable and we have assumed an isoscalar target.

Fig. 4: The coordinate system, in the laboratory frame, used to describe the directions of the leptons with respect to the incoming photon momentum.

Fig. 5: For $\theta_+ = 24^\circ$, $\theta_- = 2^\circ$, $\phi = 180^\circ$ and $E_\gamma = 150$ GeV the polarization $<\lambda_+>$ and the asymmetry A [Eqs. (25) and (26)] in % as a function of E_-. The continuous and broken lines correspond to $E_+ = 5$ GeV and $E_+ = 10$ GeV respectively.

Fig. 6: For $\theta_+ = 24^\circ$, $\theta_- = 2^\circ$, $\phi = 180^\circ$ and $E_\gamma = 200$ GeV the polarization $<\lambda_+>$ and the asymmetry A [Eqs. (25) and (26)] in % as a function of E_-. The continuous and broken lines correspond to $E_+ = 5$ GeV and $E_+ = 10$ GeV respectively.

Fig. 7: For $\theta_+ = 15^\circ$, $\theta_- = 5^\circ$, $\phi = 180^\circ$ and $E_\gamma = 150$ GeV the polarization $<\lambda_+>$ and the asymmetry A [Eqs. (25) and (26)]

in % as a function of E_-. The continuous and broken lines correspond to $E_+ = 5$ GeV and $E_+ = 10$ GeV respectively.

Fig. 8: For $\theta_+ = 15^o$, $\theta_- = 5^o$, $\phi = 180^o$ and $E_\gamma = 200$ GeV the polarization $\langle\lambda_+\rangle$ and the asymmetry A [Eqs. (25) and (26)] in % as a function of E_-. The continuous and broken lines correspond to $E_+ = 5$ GeV and $E_+ = 10$ GeV respectively.

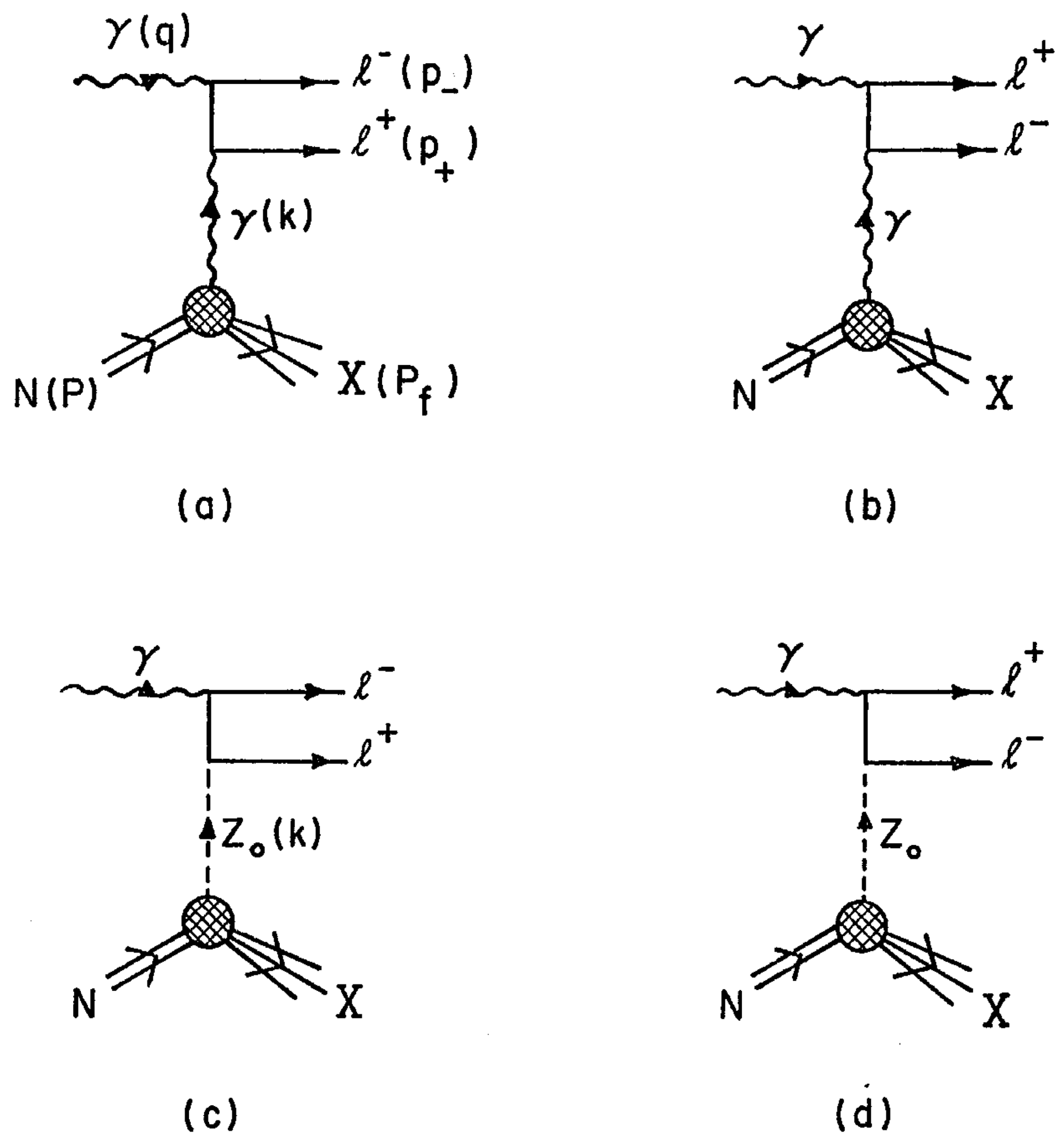

Fig. 1

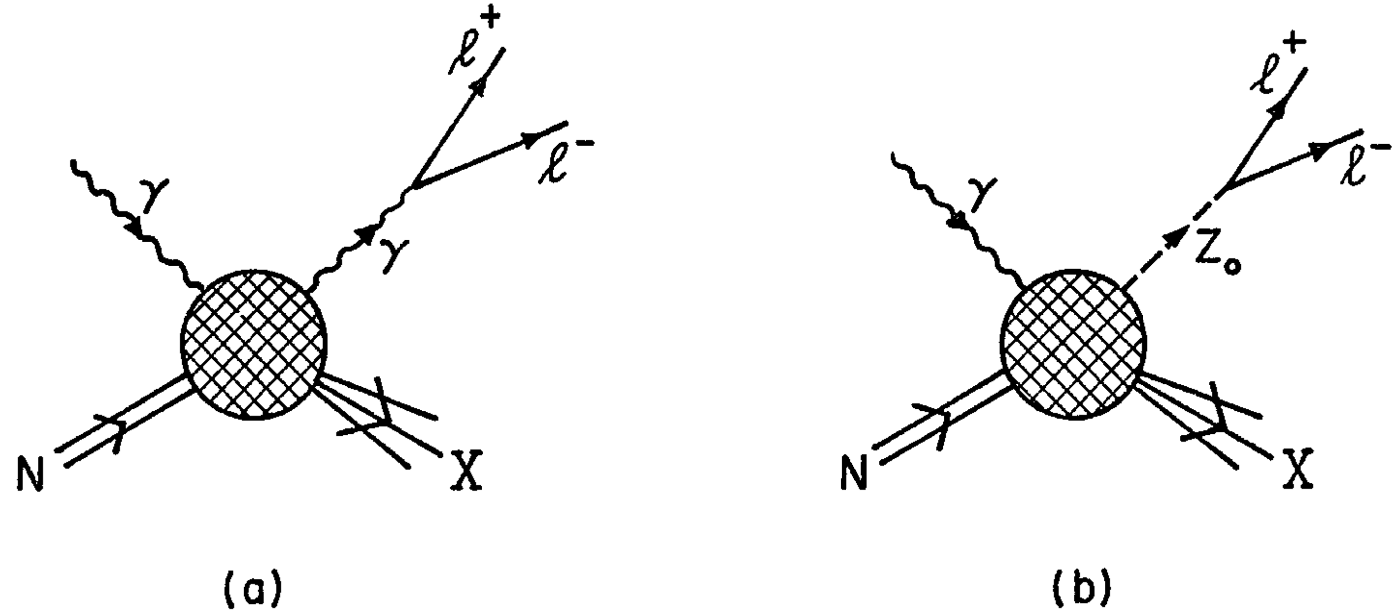

Fig. 2

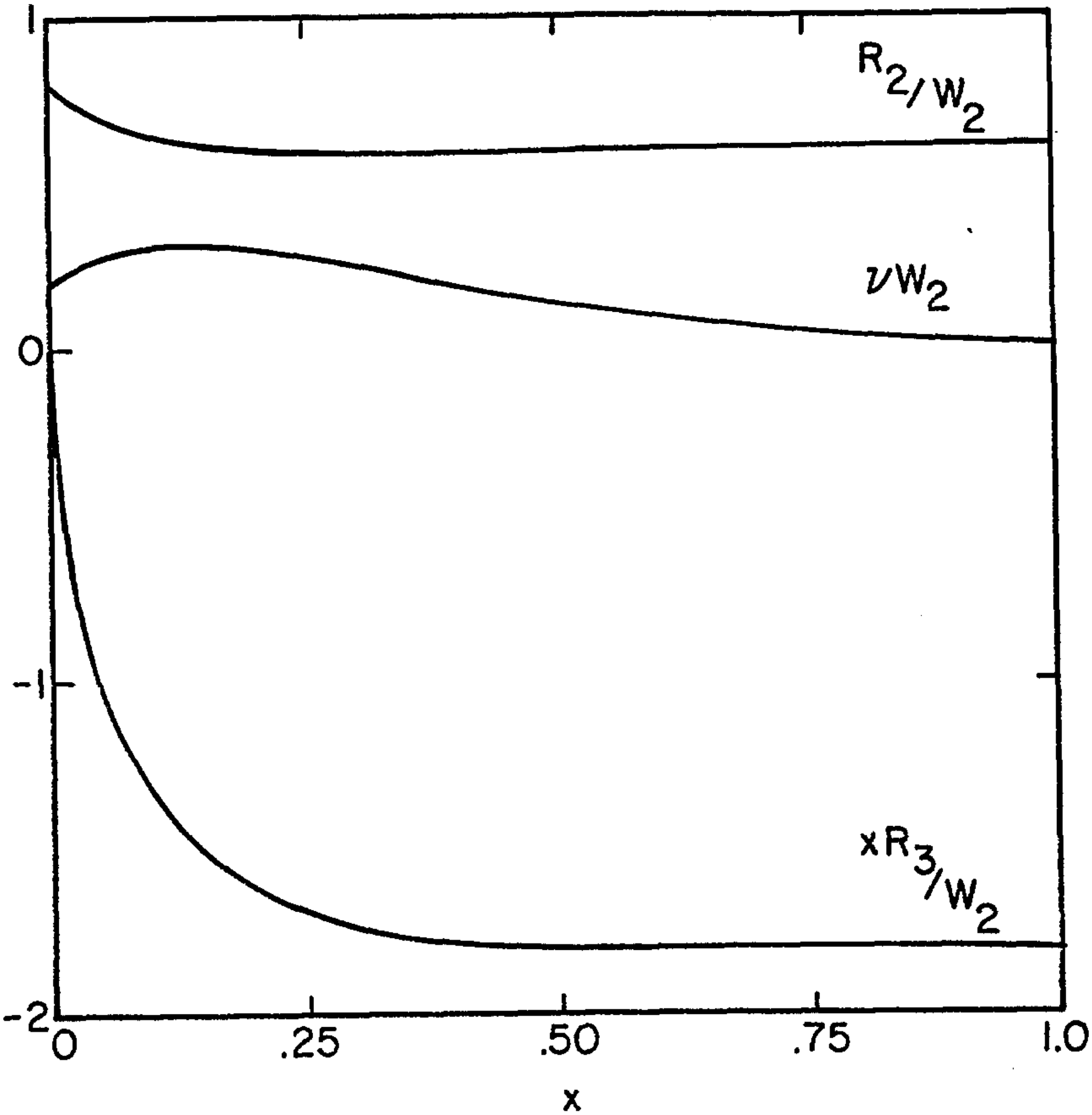

Fig. 3

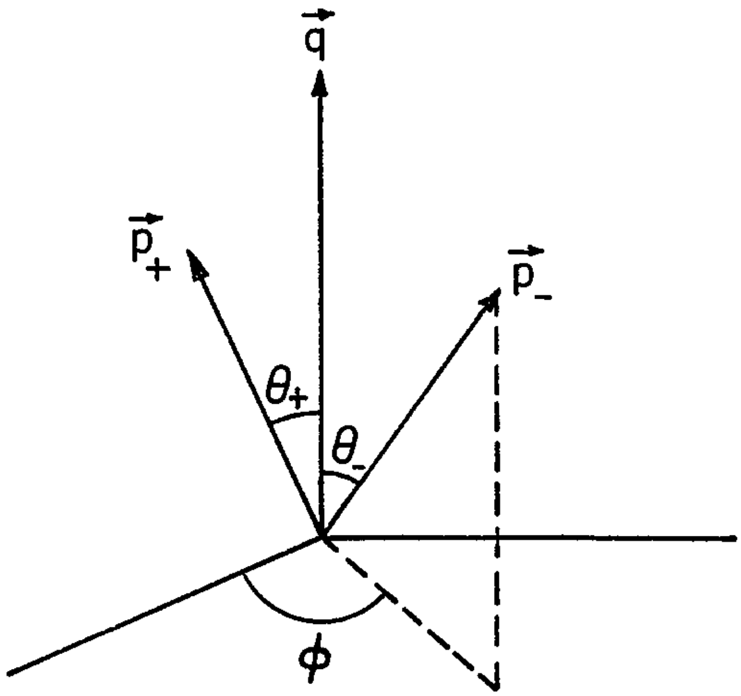

Fig. 4

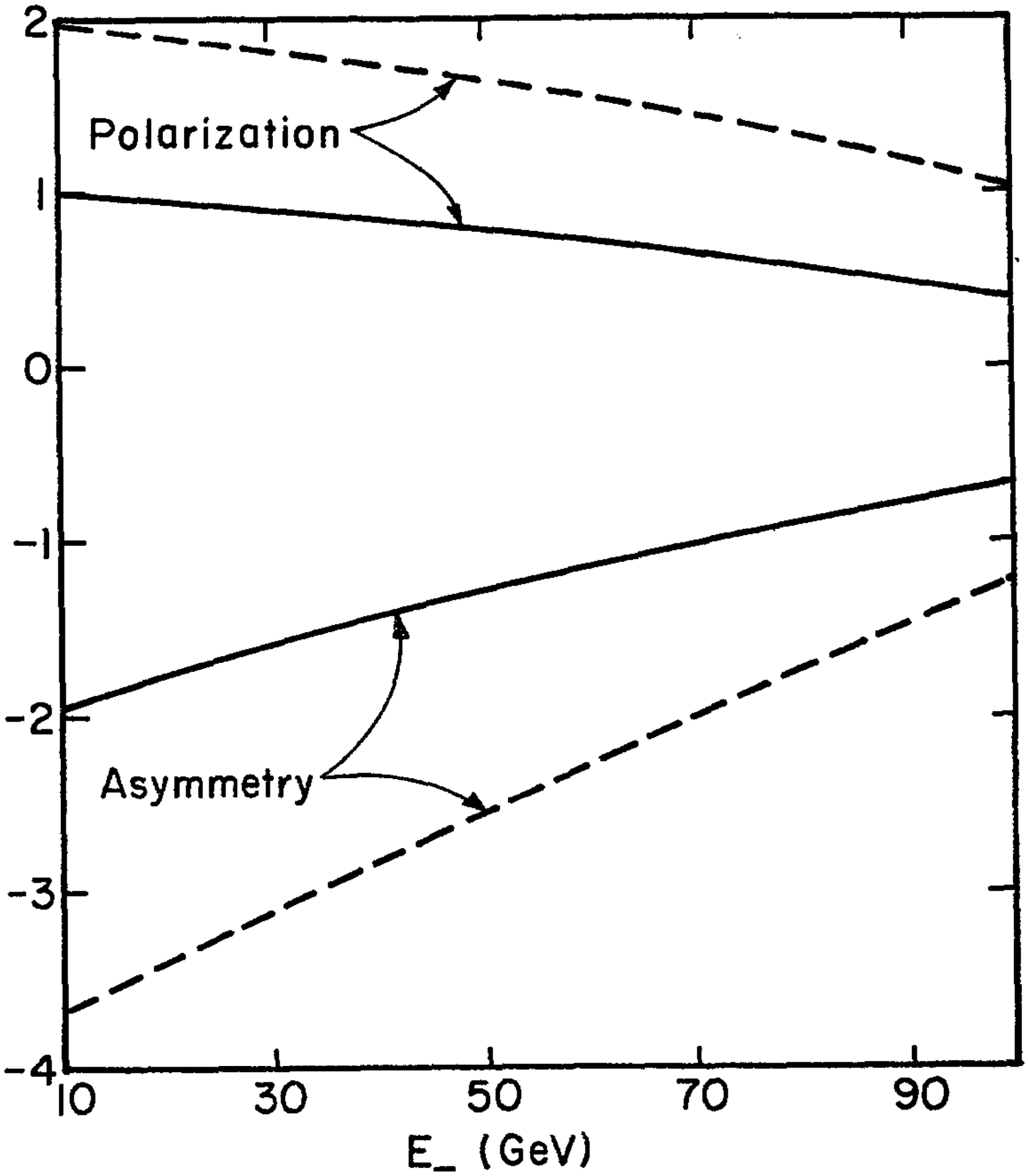

Fig. 5

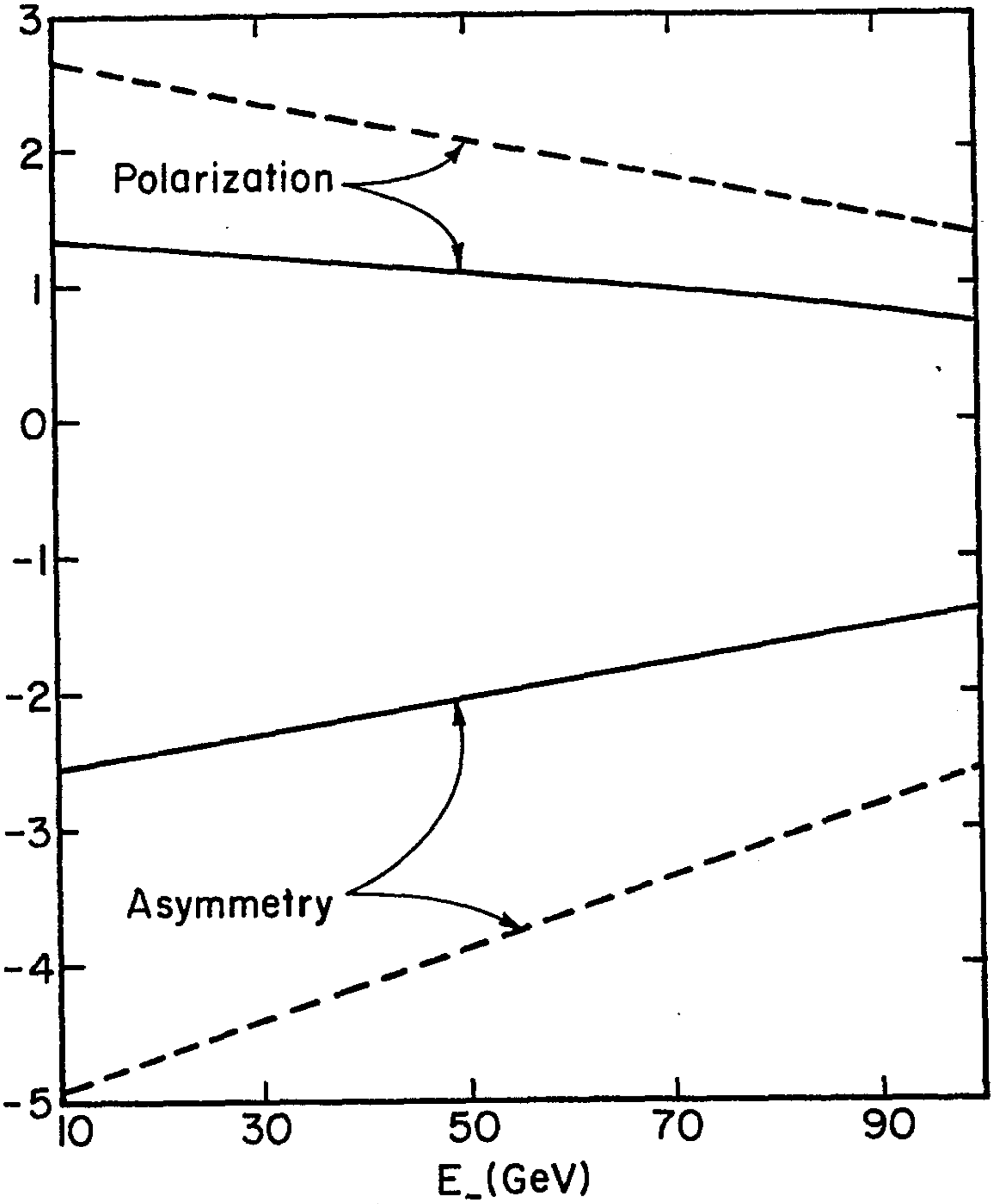

Fig. 6

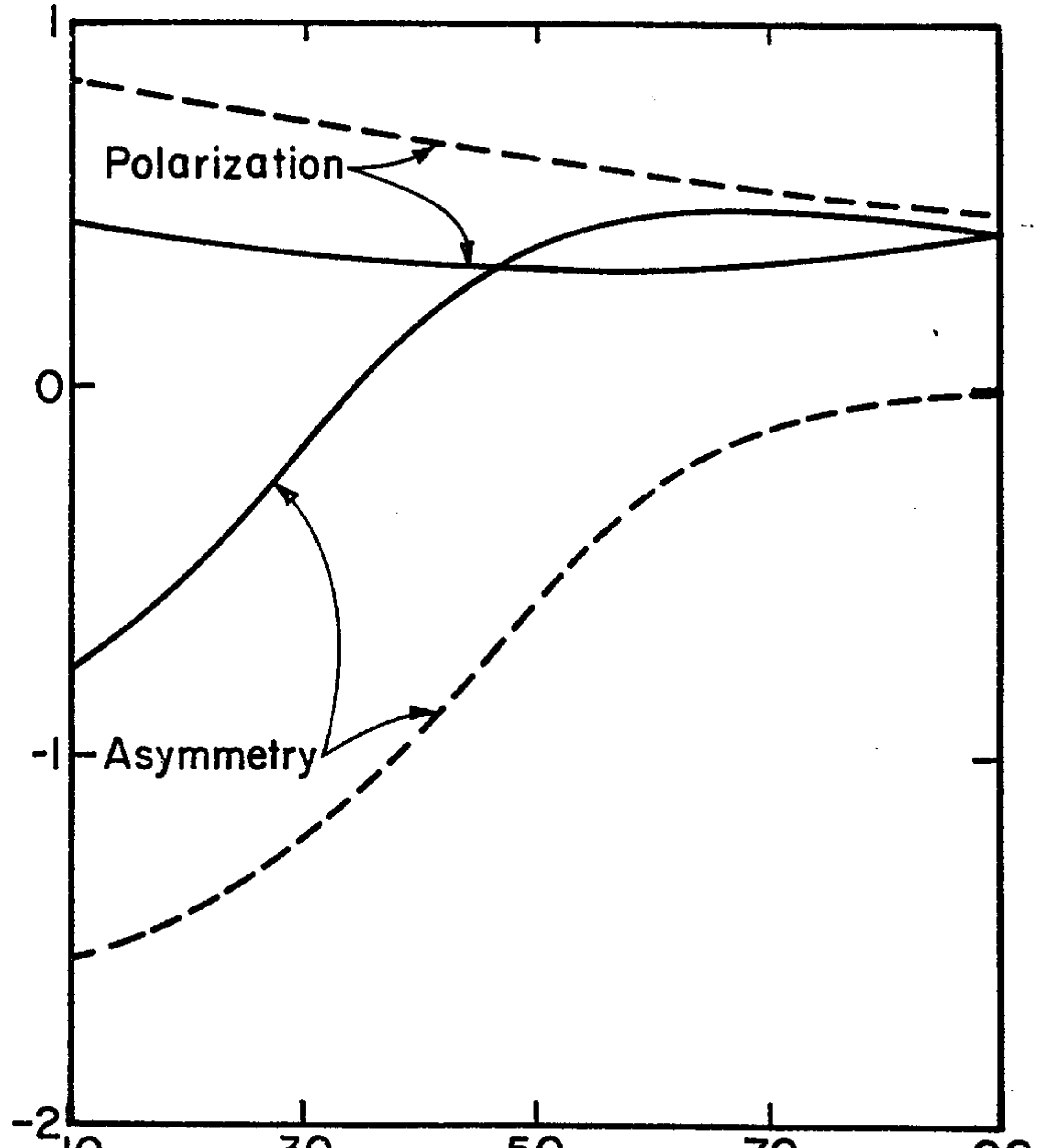

Fig. 7

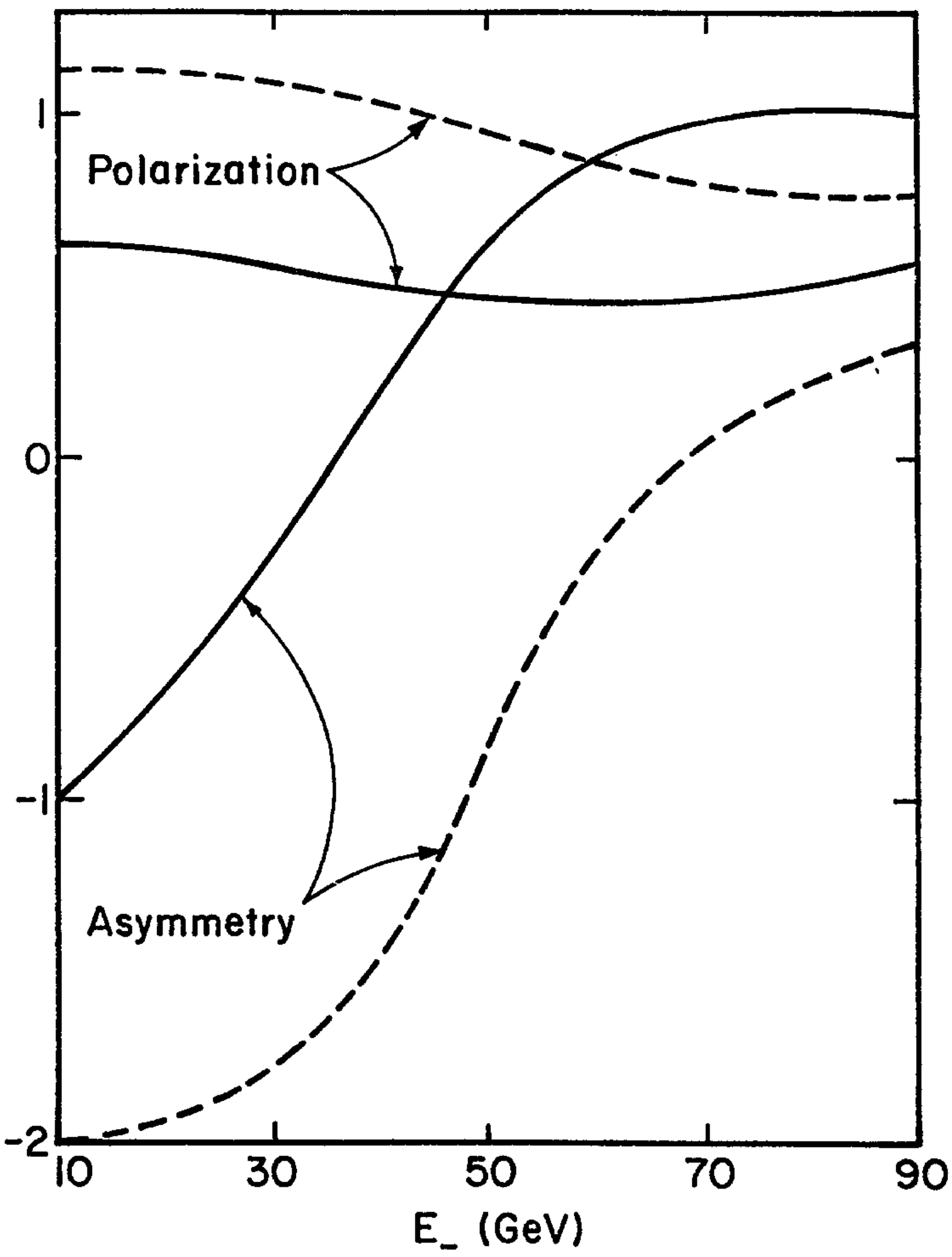

Fig. 8